**Name:** _____
**Value:** 4

I. **Multiple Choice.** Choose the **one** best answer.

1. Which of the following is correct?
   a) $T1(X) = 2X^2$ dominates $T2(X) = 2X$
   b) $T2(X) = 2X$ dominates $T1(X) = 2X^2$
   c) neither $T1(X) = 2X^2$ nor $T2(X) = 2X$ dominates the other

2. Which of the following is correct?
   a) $T1(X) = 5X^2$ dominates $T2(X) = 2X^3$
   b) $T2(X) = 2X^3$ dominates $T1(X) = 5X^2$
   c) neither $T1(X) = 5X^2$ nor $T2(X) = 2X^3$ dominates the other

3. Which of the following is correct?
   a) $T1(Y) = 55Y^{15} + 3Y^4 + 7500$ dominates $T2(Y) = Y^{16} + 2$
   b) $T2(Y) = Y^{16} + 2$ dominates $T1(Y) = 55Y^{15} + 3Y^4 + 7500$
   c) neither $T1(Y) = 55Y^{15} + 3Y^4 + 7500$ nor $T2(Y) = Y^{16} + 2$ dominates the other

4. Which of the following is correct?
   a) $T1(Z) = (Z + 3)(Z + 5)$ dominates $T2(Z) = 250Z$
   b) $T2(Z) = 250Z$ dominates $T1(Z) = (Z + 3)(Z + 5)$
   c) neither $T1(Z) = (Z + 3)(Z + 5)$ nor $T2(Z) = 250Z$ dominates the other

5. Which of the following is correct?
   a) $T1(Z) = (Z + 7)(Z + 9)$ dominates $T2(Z) = 2Z^3$
   b) $T2(Z) = 2Z^3$ dominates $T1(Z) = (Z + 7)(Z + 9)$
   c) neither $T1(X) = (Z + 7)(Z + 9)$ nor $T2(Z) = 2Z^3$ dominates the other

6. Which of the following is correct?
   a) $T1(N) = 37N^2$ dominates $T2(N) = N \log N$
   b) $T2(N) = N \log N$ dominates $T1(N) = 37N^2$
   c) neither $T1(N) = 37N^2$ nor $T2(N) = N \log N$ dominates the other

7. Which of the following is correct?
   a) $T1(N) = 37N^2$ dominates $T2(N) = N^2 \log N$
   b) $T2(N) = N^2 \log N$ dominates $T1(N) = 37N^2$
   c) neither $T1(N) = 37N^2$ nor $T2(N) = N^2 \log N$ dominates the other

8. Which of the following is correct?
   a) $T1(N) = \log_2 N$ dominates $T2(N) = \log_3 N$
   b) $T2(N) = \log_3 N$ dominates $T1(N) = \log_2 N$
   c) neither $T1(N) = \log_2 N$ nor $T2(N) = \log_3 N$ dominates the other

9. Which of the following is correct?
   a) $T1(N) = 84$ dominates $T2(N) = \log_9 N$
   b) $T2(N) = \log_9 N$ dominates $T1(N) = 84$
   c) neither $T1(N) = 84$ nor $T2(N) = \log_9 N$ dominates the other

10. Which of the following is correct?
    a) $T1(X) = \frac{3X+2}{X}$ dominates $T2(X) = 766X$
    b) $T2(X) = 766X$ dominates $T1(X) = \frac{3X+2}{X}$
    c) neither $T1(X) = \frac{3X+2}{X}$ nor $T2(X) = 766X$ dominates the other

11. Which of the following is correct?
    a) $T1(X) = \frac{X^4+X^2-17}{X^3}$ dominates $T2(X) = X^2$
    b) $T2(X) = X^2$ dominates $T1(X) = \frac{X^4+X^2-17}{X^3}$
    c) neither $T1(X) = \frac{X^4+X^2-17}{X^3}$ nor $T2(X) = X^2$ dominates the other

12. Which of the following is correct?
    a) $T1(W) = W^9$ dominates $T2(W) = 9^W$
    b) $T2(W) = 9^W$ dominates $T1(W) = W^9$
    c) neither $T1(W) = W^9$ nor $T2(W) = 9^W$ dominates the other

13. Which of the following is correct?
    a) $T1(W) = W!$ dominates $T2(W) = 67^W$
    b) $T2(W) = 67^W$ dominates $T1(W) = W!$
    c) neither $T1(W) = W!$ nor $T2(W) = 67^W$ dominates the other

14. Which of the following is correct?
    a) $T1(V) = V^V$ dominates $T2(V) = 25^V$
    b) $T2(V) = 25^V$ dominates $T1(V) = V^V$
    c) neither $T1(V) = V^V$ nor $T2(V) = 25^V$ dominates the other

15. Which of the following is correct?
    a) $T1(X, Y) = 2X^3Y$ dominates $T2(X, Y) = 3XY$
    b) $T2(X, Y) = 3XY$ dominates $T1(X, Y) = 2X^3Y$
    c) neither $T1(X, Y) = 2X^3Y$ nor $T2(X, Y) = 3XY$ dominates the other

16. Which of the following is correct?
    a) $T1(X, Y) = X^2 + 75$ dominates $T2(X, Y) = 7Y$
    b) $T2(X, Y) = 7Y$ dominates $T1(X, Y) = X^2 + 75$
    c) neither $T1(X, Y) = X^2 + 75$ nor $T2(X, Y) = 7Y$ dominates the other

17. Which of the following is correct?
    a) $T1(V, W) = W^3 + W + 74$ dominates $T2(V, W) = V + 18$
    b) $T2(V, W) = V + 18$ dominates $T1(V, W) = W^3 + W + 74$
    c) neither $T1(V, W) = W^3 + W + 74$ nor $T2(V, W) = V + 18$ dominates the other

II. Determine the big-$O$ measure for each of the functions in the lower table. Choose from the various big-$O$ measures in the upper table.

| | | | | | |
|---|---|---|---|---|---|
| A. $O(1)$ | B. $O(X)$ | C. $O(\log X)$ | D. $O(X \log X)$ | E. $O(X^2)$ | F. $O(X^3)$ |
| G. $O(Y)$ | H. $O(\log Y)$ | I. $O(Y \log Y)$ | J. $O(Y^2)$ | K. $O(Y^3)$ | L. $O(Y^{15})$ |
| M. $O(Y^{16})$ | N. $O(Z)$ | O. $O(Z^2)$ | P. $O(Z^3)$ | Q. $O(\log N)$ | R. $O(N)$ |
| S. $O(N \log N)$ | T. $O(N^2)$ | U. $O(N^2 \log N)$ | V. $O(W^3)$ | W. $O(W^9)$ | X. $O(9^W)$ |
| Y. $O(W!)$ | Z. $O(67^W)$ | AA. $O(V)$ | BB. $O(V^V)$ | CC. $O(25^V)$ | DD. $O(XY)$ |
| EE. $O(X^3Y)$ | | | | | |

| | | | |
|---|---|---|---|
| 18. | $T1(X) = 2X^2$ | 19. | $T2(X) = 2X$ |
| 20. | $T1(X) = 5X^2$ | 21. | $T2(X) = 2X^3$ |
| 22. | $T1(Y) = 55Y^{15} + 3Y^4 + 7500$ | 23. | $T2(Y) = Y^{16} + 2$ |
| 24. | $T1(Z) = (Z + 3)(Z + 5)$ | 25. | $T2(Z) = 250Z$ |
| 26. | $T1(Z) = (Z + 7)(Z + 9)$ | 27. | $T2(Z) = 2Z^3$ |
| 28. | $T1(N) = 37N^2$ | 29. | $T2(N) = N \log N$ |
| 30. | $T1(N) = 37N^2$ | 31. | $T2(N) = N^2 \log N$ |
| 32. | $T1(N) = \log_2 N$ | 33. | $T2(N) = \log_3 N$ |
| 34. | $T1(N) = 84$ | 35. | $T2(N) = \log_9 N$ |
| 36. | $T1(X) = \frac{3X+2}{X}$ | 37. | $T2(X) = 766X$ |
| 38. | $T1(X) = \frac{X^4+X^2-17}{X^3}$ | 39. | $T2(X) = X^2$ |
| 40. | $T1(W) = W^9$ | 41. | $T2(W) = 9^W$ |
| 42. | $T1(W) = W!$ | 43. | $T2(W) = 67^W$ |
| 44. | $T1(V) = V^V$ | 45. | $T2(V) = 25^V$ |
| 46. | $T1(X, Y) = 2X^3Y$ | 47. | $T2(X, Y) = 3XY$ |
| 48. | $T1(X, Y) = X^2 + 75$ | 49. | $T2(X, Y) = 7Y$ |
| 50. | $T1(V, W) = W^3 + W + 74$ | 51. | $T2(V, W) = V + 18$ |

III. Rank the following big-$O$ measures from greatest to least. (Recall that $O(f) > O(g)$ if and only if $f$ dominates $g$.)

a. $O(N)$

b. $O(N^3)$

c. $O(4^N)$

d. $O(\log_4 N)$

e. $O(\log_5 N)$

f. $O(N^2)$

g. $O(1)$

h. $O(N \log_3 N)$

i. $O(N^2 \log_3 N)$

| 52. | 53. | 54. | 55. | 56. | 57. | 58. | 59. | 60. |
|---|---|---|---|---|---|---|---|---|

IV. Classify each big-$O$ measure in the lower table using the classfications from the upper table. Choose the one best classification.

| A. constant | B. linear | C. quadratic | D. cubic |
|---|---|---|---|
| E. logarithmic | F. polynomial | G. exponential | |

| 61. | $O(N)$ | 62. | $O(N^3)$ | 63. | $O(4^N)$ |
|---|---|---|---|---|---|
| 64. | $O(\log_4 N)$ | 65. | $O(\log_5 N)$ | 66. | $O(N^2)$ |
| 67. | $O(1)$ | 68. | $O(N \log_3 N)$ | 69. | $O(N^2 \log_3 N)$ |

V. Determine the order of each function in the lower table using the classfications from the upper table. Choose the one best classification.

| A. $O(1)$ | B. $O(\log N)$ | C. $O(N)$ | D. $O(N^2)$ |
|---|---|---|---|
| E. $O(N^3)$ | F. $O(2^N)$ | G. $O(3^N)$ | H. $O(17^N)$ |

| 70. | $T(N) = 3N^2 + N$ | 71. | $T(N) = 55N^3 + 77N^2 + 99$ | 72. | $T(N) = 2^N N^2$ |
|---|---|---|---|---|---|
| 73. | $T(N) = 7501$ | 74. | $T(N) = \log N + 46N$ | 75. | $T(N) = 3^{(N+1)}$ |
| 76. | $T(N) = \frac{N \log N}{2+N}$ | 77. | $T(N) = \frac{3N^4+4N^3}{5N^2+N}$ | 78. | $T(N) = \frac{17^N}{N^2}$ |

VI. Determine the big-$O$ measure for each of the functions in the lower table. Classify each big-$O$ measure using the classfications from the upper table. Choose the one best classification.

| | | | |
|---|---|---|---|
| A. constant | B. linear | C. quadratic | D. cubic |
| E. logarithmic | F. polynomial | G. exponential | |

| | | | | | |
|---|---|---|---|---|---|
| 79. | $T(N) = 3N^2 + N$ | 80. | $T(N) = 55N^3 + 77N^2 + 99$ | 81. | $T(N) = 2^N N^2$ |
| 82. | $T(N) = 7501$ | 83. | $T(N) = \log N + 46N$ | 84. | $T(N) = 3^{(N+1)}$ |
| 85. | $T(N) = \frac{N \log N}{2+N}$ | 86. | $T(N) = \frac{3N^4+4N^3}{5N^2+N}$ | 87. | $T(N) = \frac{17^N}{N^2}$ |

VII. Using big-$O$ notation, estimate the runnning time of each of the following algorithms. You may assume that all variables are of type `int`. Choose the one best big-$O$ measure from the table below.

| | | |
|---|---|---|
| A. $O(1)$ | B. $O(\log X)$ | C. $O(X)$ |
| D. $O(X \log X)$ | E. $O(X^2)$ | F. $O(X^3)$ |
| G. $O(X!)$ | H. $O(X^X)$ | I. $O(X^2Y)$ |

88.
```
1  for (i = 1; i <= X; i++)
2    for (j = 1; j <= X; j++)
3      for (k = 1; k <= X; k++)
4      {
5        // Five assignment instructions
6      }
```

89.
```
1  for (i = 10; i <= X; i++)
2  {
3    // Two assignment instructions
4    for (j = 15; j <= X; j++)
5    {
6      for (k = 1; k <= X; k++)
7      {
8        // Five assignment instructions
9      }
10     // Seven assignment instructions
11   }
12 }
```

90.
```
for (i = 1; i <= X; i++)
{
  for (j = 1; j <= X; j++)
  {
    // Twenty assignment instructions
  }
  for (j = 1; j <= X; j++)
    if (j % 2 == 1)
      for (k = 1; k <= X; k++)
      {
        // Five assignment instructions
      }
}
```

91.
```
for (i = 1; i <= X; i++)
{
  for (j = i; j <= X; j++)
  {
    // Six assignment instructions
  }
  if (i % 2 == 1)
  {
    // Four assignment instructions
  }
}
```

92.
```
for (i = 1; i <= X; i++)
  for (j = 1; j <= Y; j++)
    for (k = 1; k <= X; k++)
    {
      // Two assignment statements
    }
```

93.
```
i = 1;
while (i <= X)
{
  // Three assignment instructions
  j = 17;
  while (j <= 100)
  {
    // Two assignment instructions
    j++;
  }
  i++;
}
```

94.
```
i = X;
do
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j++;
  }
  i--;
} while (i >= 1);
```

95.
```
i = 1;
while (i <= X)
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j = j * 2;
  }
  i++;
}
```

96.
```
i = X;
while (i >= 1)
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j = j + 2;
  }
  i = i / 3;
}
```

97.
```
void RecA(int X)
{
  // Some task requiring constant time
  if (X > 0)
    RecA(X - 1);
}
```

98.
```
void RecB(int X)
{
  int i;

  for (i = 1; i <= X; i++)
  {
    // Some task requiring constant time
  }
  if (X > 1)
    RecB(X - 1);
}
```

99.
```
void RecC(int X)
{
  int i;

  for (i = 1; i <= X; i++)
  {
    // Some task requiring constant time
  }
  if (X > 1)
    RecC(X / 2);
}
```

100.
```
void RecD(int X)
{
  int i;

  // Some task requiring constant time
  for (i = 1; i <= X; i++)
    if (X > 1)
      RecD(X - 1);
}
```